WOBURN CHALLENGE

2015-16 Online Round 2

Friday, December 11th, 2015

Senior Division Problems

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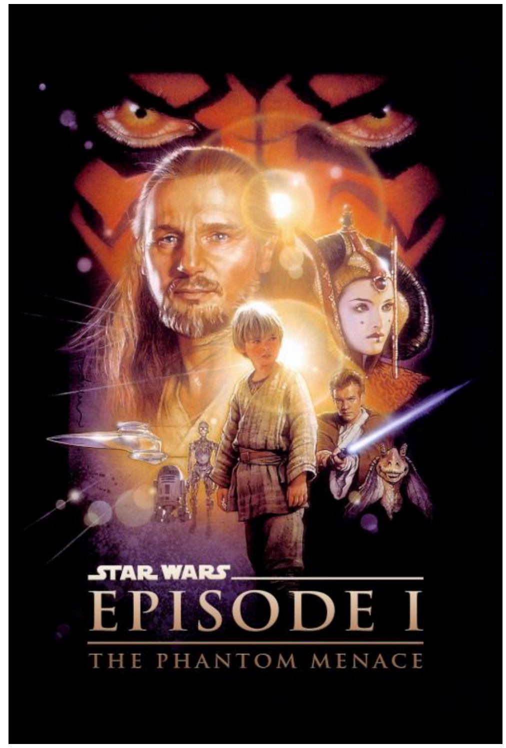
Problem I: The Phantom Menace

10 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc152s1*

"There is no doubt. The mysterious coder was a Sith."
"Always two there are, no more, no less. A master and a n00b."
"But which was destroyed, the master or the n00b?"



The exciting climax of the highly-anticipated 1999 *Star Wars* prequel features four distinct action sequences occurring simultaneously:

1. Jar Jar Binks leads the Gungan army in a diversionary battle against the Trade Federation's droid army on the plains of Naboo.
2. Queen Amidala and her security force storm the palace in Theed in an attempt to locate and arrest Viceroy Nute Gunray.
3. Obi-Wan Kenobi and Qui-Gon Jinn have a lightsaber duel against the Sith lord Darth Maul.
4. Anakin Skywalker pilots a Naboo starfighter into orbit amidst a massive dogfight, attempting to take out the Trade Federation's droid control ship.

During this time, the film needs to cut back and forth between these settings. In particular, there are N ($1 \le N \le 10{,}000$) scenes to fill, with each scene observing one of the 4 settings. Additionally, no two consecutive scenes may be assigned to the same setting.

Some of the scenes have already been assigned to settings, while others haven't been. The status of the i-th scene is indicated by the value of S_i ($0 \le S_i \le 4$). If $S_i = 0$, then the scene is currently unassigned, and if $S_i > 0$, then the scene must observe setting S_i.

Your job is to assign each of the unassigned scenes to one of the 4 settings, such that no two consecutive scenes are assigned to the same one. It's guaranteed that this will be possible. However, it may be possible in multiple ways, in which case you must choose the one that minimizes the "numeric representation" of the scenes. The numeric representation consists of concatenating the values of the N scenes' assigned settings, in order from 1 to N, and treating the result as an integer (with each digit in the range 1..4).

Input Format

The first line of input consists of a single integer N. The second line consists of N space-separated integers S_1 to S_N.

Output Format

Output on a single line the numeric representation of your finalized scenes.

Sample Input

```
8
0 4 1 0 0 1 3 0
```

Sample Output

```
14123131
```

Problem II: Attack of the Clones

20 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc152s2*

"You know I don't like it when you do that."
"Sorry, master. I forgot that you don't like coding."
"I don't mind coding, but what you're doing is suicide!"

Upon thwarting an attempt on Senator Amidala's life, Obi-Wan Kenobi and Anakin Skywalker must pursue the bounty hunter Zam Wesell through the bustling streets of Coruscant in order to apprehend and question her. Unfortunately, they've already lost her trail! Fortunately, they have some ideas of where she might be hiding.

The city of Coruscant is laid out in a regular grid of intersections. There are many parallel avenues running North-South, numbered from West to East starting from 1. Similarly, there are many parallel streets running East-West, numbered from North to South starting from 1. The position where street s and avenue a intersect can be denoted as (s, a).

All of the streets and avenues are one-way, and even in this emergency, the Jedi must respect the law. Every avenue may only be travelled along towards the South. However, the directions of the streets alternate – every odd-numbered street may only be travelled along towards the East, while every even-numbered street may only be travelled along towards the West. It takes 1 minute to travel from a certain intersection to an adjacent one (either to the South, East, or West, while obeying the traffic laws).

The Jedi are currently in the Senate building at intersection (1, 1). There are N ($1 \le N \le 200{,}000$) potential locations at which Zam might be hiding, numbered from 1 to N in decreasing order of likelihood, with the i-th one being intersection (S_i, A_i) ($1 \le S_i, A_i \le 40{,}000$). None of the N locations is (1, 1), and no two of them are at the same intersection.

A decision must be made on how many of the locations to try. As such, for every i in the range $1..N$, you must determine the minimum amount of time (in minutes) that it would take for the Jedi to theoretically visit all of the first i locations (in any order), starting from (1, 1).

In test cases worth 75% of the points, $N \le 2000$.
In a subset of those cases worth 30% of the points, $N \le 100$, $S_i \le 100$, and $A_i \le 100$.

Input Format

The first line of input consists of a single integer N.
The next N lines each consist of two space-separated integers S_i and A_i, for $i = 1..N$.

Output Format

The output should consist of N lines, where the i-th line is a single integer representing the minimum amount of time (in minutes) that it would take for the Jedi to theoretically visit all of the first i locations (in any order), starting from (1, 1).

Sample Input

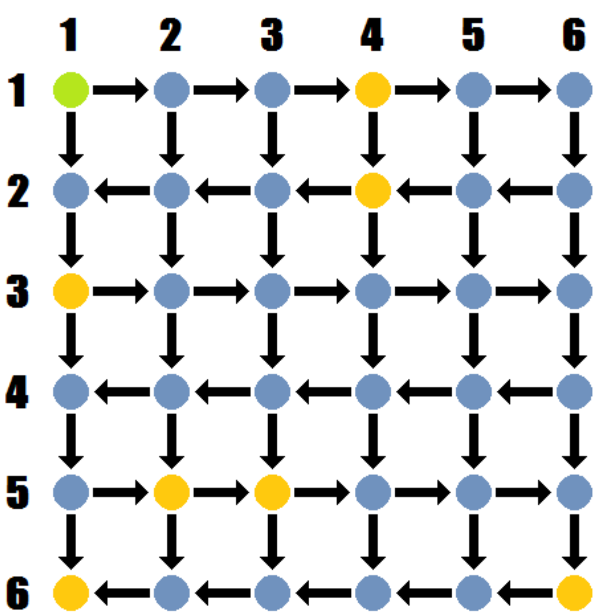
```
7
5 3
5 2
2 4
6 1
1 4
3 1
6 6
```

Sample Output

```
6
6
10
13
13
15
21
```

Explanation

The following diagram depicts the scenario in the sample input. Green represents the senate building, blue represents a normal intersection, and yellow represents potential locations at which Zam might be hiding.



Problem III: Revenge of the Sith

30 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc152s3*

"If you're not with me, then you're my enemy!"
"Only a Sith deals in absolute values."

Obi-Wan Kenobi and Anakin Skywalker, once the best of friends, find themselves about to have a vicious duel on a volcanic planet in the Mustafar system. Their fight will take place in the Separatists' hideout, a base consisting of N ($2 \le N \le 45{,}678$) chambers and M ($0 \le M \le 234{,}567$) corridors. The i-th corridor connects chambers A_i and B_i, and can be traversed in either direction. No pair of chambers are directly connected by multiple corridors.

There's just one hold up – Obi-Wan and Anakin must find each other first. Anakin is waiting in a random chamber, and Obi-Wan is similarly about to land in a random chamber. With each of these locations chosen uniformly at random from the set of possible chambers, there are N^2 possible, equally-likely ordered pairs of starting chambers.

As soon as Obi-Wan lands in his chamber, he and Anakin will start looking for one another. Exactly once every minute, each of them will travel along a random corridor connected to their current chamber (chosen uniformly at random from the set of such corridors), unless their current chamber isn't connected to any corridors, in which case they'll stay where they are. If the two of them find themselves in the same chamber during any given minute, they'll commence their duel. Otherwise, they'll continue this process infinitely (Jedi do live for a long time). Note that, with the power of the Force, they each travel through their chosen corridor extremely quickly every minute – therefore, they can never meet up in a corridor, even if they both travel through the same one at the same time (in opposite directions).

What's the probability that Obi-Wan and Anakin will eventually meet up and actually have their duel?

In cases worth 40% of the points, $N \le 1234$ and $M \le 4567$.
In a subset of those cases worth 20% of the points, $N \le 34$ and $M \le 456$.

Input Format

The first line of input consists of two space-separated integers N and M.
The next M lines each consist of two space-separated integers A_i and B_i, for 1 = 1..M.

Output Format

Output a single real number between 0 and 1, the probability that Obi-Wan and Anakin will eventually duel.
Your answer must have an absolute error of no more than 10^{-6}.

Sample Input

```
6 7
2 3
2 4
2 5
3 4
3 5
4 5
6 1
```

Sample Output

```
0.5
```

Explanation

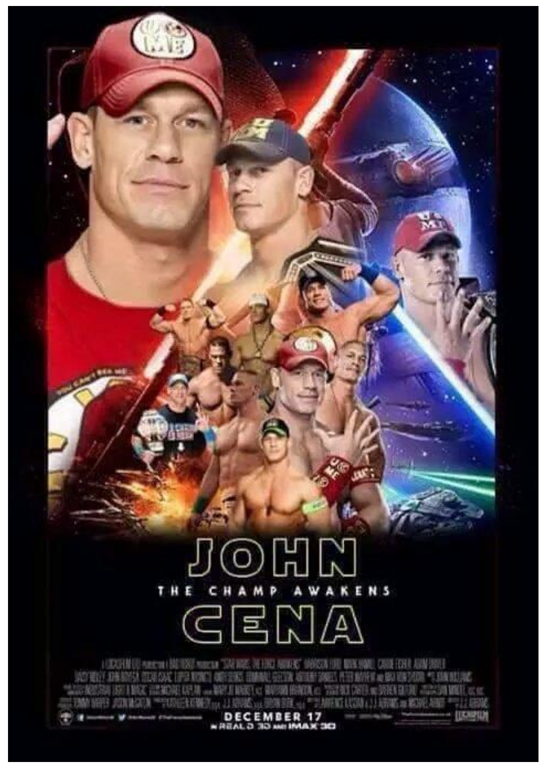
There are 36 possible, equally-likely ordered pairs of starting chambers for Obi-Wan and Anakin. It so happens that for 18 of them, the probability of an eventual encounter is 100% (including the pairs (1, 1) and (2, 4)), while for the other 18, it's 0% (including the pairs (1, 6) and (5, 6)).

Problem VII: The CHAMP Awakens

40 Points / Time Limit: 3.00s / Memory Limit: 256M *Submit online: wcipeg.com/problem/wc152s4*

"The Force is strong in my family..."
"My father has it."
"I have it."
"My sister has it."
"There's one other person with that power..."

AND HIS NAME IS JOHN CENA



With the Death Star reduced to stardust, the evil Galactic Empire has been forced to pull out all stops in their fight against the rebels. N ($1 \le N \le 400{,}000$) heavily-armed Imperial starships have been deployed to fly across the galaxy, destroying all rebel resistance in their paths. All hope would seem lost, but the rebels have a powerful new Jedi ally on their side… and his name is John Cena.

John Cena has devised a plan to deal massive amounts of damage to the Imperial squadron. Upon studying stolen flight plans, he's realized that all of the enemy ships will always stay within a single plane of space. Then, treating it as a 2D Cartesian plane, he's set up a "blast zone" – a rectangle with its bottom-left corner at coordinates (X_1, Y_1) and its top-right corner at (X_2, Y_2) ($-10^5 \le X_1 < X_2 \le 10^5$, $-10^5 \le Y_1 < Y_2 \le 10^5$). He has access to a device which can set off a powerful electrical charge throughout the blast zone, damaging ships within it (including ones right on its border)!

As mentioned, John Cena has access to the Imperial flight plans, which happen to be quite simple. The i-th ship will initially be located at coordinates (s_{xi}, s_{yi}) ($-10^5 \le s_{xi}$, $s_{yi} \le 10^5$), and will then fly in a straight line at a constant velocity of d_{xi} horizontal units and d_{yi} vertical units per second ($-10^5 \le d_{xi}$, $d_{yi} \le 10^5$). No ship is stationary – that is, $|d_{xi}| + |d_{yi}| > 0$. Additionally, multiple ships may occupy the same location at any point in time.

Now, John Cena's battle plan will consist of M ($1 \le M \le 400{,}000$) steps. The i-th step can be of one of two types, given by the value of A_i ($1 \le A_i \le 2$):

- $A_i = 1$: First, wait B_i ($0 \le B_i \le 5000$) seconds after the previous step, and then set off a charge to deal C_i ($1 \le C_i \le 10^5$) damage to each ship which is currently within the blast zone (inclusively)
- $A_i = 2$: To assess success, determine the total damage dealt so far to ships $B_i..C_i$ ($1 \le B_i \le C_i \le N$)

John Cena's time is now, but can you help him determine the results of each of his steps of type 2?

Note: In cases worth 20% of the points, $N \le 500$ and $M \le 2000$.

Input Format

The first line of input consists of two space-separated integers N and M.
The second line consists of four space-separated integers X_1, Y_1, X_2, and Y_2
The next N lines each consist of four space-separated integers s_{xi}, s_{yi}, d_{xi}, and d_{yi}, for $i = 1..N$.
The next M lines each consist of three space-separated integers A_i, B_i, and C_i, for $i = 1..M$.

Output Format

For each step in the input where $A_i = 2$, output a single integer on a separate line – the total amount of damange dealt so far to ships in the range specified by B_i and C_i. Note that each result may not fit within a 32-bit signed integer.

Sample Input

```
3 14
-2 -1 3 3
7 2 -2 0
-1 -6 1 2
-1 0 0 3
1 0 5
1 1 7
2 1 2
2 1 3
1 1 1
1 0 1
2 1 3
1 1 30
1 2 6
1 1 1000
2 1 1
2 2 2
2 3 3
2 2 3
```

Sample Output

```
0
12
14
32
30
12
42
```

Explanation

The first charge is set off immediately, and the second occurs 1 second later - during both of these, only the 3rd ship is within the blast zone, so it takes 12 damage. The next 2 charges both occur 1 second after that, at which point the 3 ships are at coordinates (3, 2), (1, −2) and (−1, 6), respectively – as such, only the 1st ship is hit, sustaining 2 damage. The following charge deals 30 damage to both ships 1 and 2, while neither of the last 2 charges hit any ships.