WOBURN CHALLENGE

2015-16 Online Round 4

Friday, April 8th, 2016

Senior Division Problems

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Problem S1: Shootout

17 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc154s1*

James Bond's latest mission is not going as planned. He's suddenly found himself at one end of a long, narrow corridor which is filled with N ($1 \le N \le 200{,}000$) of Blofeld's henchmen. The i-th henchman is standing H_i ($1 \le H_i \le 10^9$) metres away from Bond along the corridor.

There are also M ($0 \le M \le 200{,}000$) doors in the corridor, all of which are initially closed, with the i-th door D_i ($1 \le D_i \le 10^9$) metres away from Bond along the corridor. Of the $N + M$ henchmen and doors, no two of them are at the same location.

The building's security system will open all of the doors in order, one after another, starting from door 1 and ending with door M. Once each door has been opened, it will stay open permanently. In order to do his best to die another day, Bond will need to quickly assess how many of the henchmen are currently in his line of fire after each door is opened. A henchman is in Bond's line of fire if there are no closed doors between them and Bond.

Fortunately, Bond brought along his personal computer to the gunfight to help with these computations. Unfortunately, he forgot to get the floppy disk containg the program from Q! As quickly as you can, for each i from 1 to M, please help Bond determine how many of the N henchmen will be in his line of fire after the first i doors have been opened.

Subtasks

In test cases worth 3/17 of the points, $N \le 100$ and $M \le 100$.
In test cases worth another 4/17 of the points, $N \le 200$ and $M \le 3000$.
In test cases worth another 4/17 of the points, $N \le 200$ and $M \le 40{,}000$.

Input Format

The first line of input consists of two space-separated integers N and M.
The next N lines each consist of a single integer H_i, for $i = 1..N$.
The next M lines each consist of a single integer D_i, for $i = 1..M$.

Output Format

Output M lines with one integer per line. The i-th line of output (for $i = 1..M$) should consist of the number of henchmen in Bond's line of fire after i doors have been opened.

Sample Input

```
5 4
2
300
4
15
1000000000
200
3
100
301
```

Sample Output

```
1
3
4
5
```

Problem S2: World Tour

23 Points / Time Limit: 6.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc154s2*

James Bond is in trouble – the fearsome Jaws is after him! Jaws will stop at nothing to find Bond and take him out (and not out to eat, though biting will be involved), and he's gotten incredibly skilled at picking up Bond's trail as he travels around the world on his assignments.

Unfortunately for Jaws, this ability to follow Bond's trail will be his undoing. Bond intends to take Jaws on a little world tour, tricking him into flying to all sorts of cities by leaving behind fake clues about where he himself is headed.

There are N ($1 \le N \le 500{,}000$) cities of interest, and in the i-th of these cities, Bond will leave a clue suggesting that he travelled from that city to a different city C_i ($1 \le C_i \le N$, $C_i \ne i$). This means that, if Jaws ever finds himself in city i, he's sure to fly to city C_i from there!

Depending on which city Jaws starts in, the sequence of flights he'll be obligated to take while following Bond's supposed trail will differ. In fact, each such sequence of flights will be infinitely long, as poor Jaws will always blindly go on chasing his adversary's scent, even if he passes through some of the cities multiple times along the way! It's a good thing he'll be able to file an expense report to get reimbursed for all of these flights afterwards (they're for business purposes, after all). That being said, over the course of his infinitely-long trip, he certainly won't visit an infinite number of *different* cities. For each of the N potential starting cities, can you count the number of different cities (including that one) which Jaws will end up visiting if he starts there and continues taking his required flights forever?

Subtasks

In test cases worth 7/23 of the points, $N \le 2000$.

Input Format

The first line of input consists of a single integer N.
The next N lines each consist of a single integer C_i, for $i = 1..N$.

Output Format

Output N lines, one integer per line. The i-th line (for $i = 1..N$) should consist of the number of different cities that Jaws will end up visiting if he starts in city i.

Sample Input

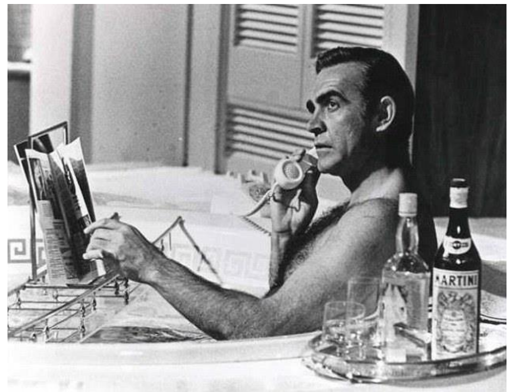
```
4
4
1
2
1
```

Sample Output

```
2
3
4
2
```

Problem S3: Coded Paper

25 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc154s3*



Bond has gotten his hands on a mysterious slip of paper. It's divided into a grid of square cells, with 2 rows and N ($1 \le N \le 10^5$) columns. The j-th cell in the i-th row has the integer $C_{i,\,j}$ ($-100 \le C_{i,\,j} \le 100$) printed in it.

Bond also has access to a curious machine which can print rectangular pieces of cardboard on demand. He can use it to print as many rectangles as he'd like, each of them having any integral dimensions of his choice. Each rectangle produced by this machine will have a single integer R ($-100 \le R \le 100$) printed on it, regardless of its size. Bond may choose to place these cardboard rectangles on top of the slip of paper such that they cover some of its cells, as long as they're aligned with the grid, they fit within the grid, and none of them overlap with one another.

He has reason to believe that the purpose of this slip of paper is actually to encode a single, secret integer, which is the maximum possible sum of visible integers after zero or more cardboard rectangles have been placed on it. This sum includes the integers written on the rectangles used, of course, and doesn't include any integers in cells which are underneath a rectangle. Can you help him crack the code by determining the maximum possible sum that can be achieved?

Subtasks

In test cases worth 2/25 of the points, $N \le 2$.
In test cases worth another 5/25 of the points, $N \le 6$.

Input Format

The first line of input consists of two space-separated integers N and R.
The second line consists of N integers, $C_{1,\,1..N}$.
The third line consists of N integers, $C_{2,\,1..N}$.

Output Format

Output a single integer, the maximum sum of visible integers that Bond can achieve by placing cardboard rectangles optimally.

Sample Input 1

```
4 -5
20 -10 -1 5
-2 3 1 -6
```

Sample Output 1

```
17
```

Explanation

Bond should place a 1×2 rectangle in the middle of the first row (covering −10 and −1), and a 1×1 rectangle in the bottom-right corner (covering −6). The visible numbers will then be 20, −5, 5, −2, 3, 1, and −5, which sum to 17.

Sample Input 2

```
2 -1
-10 -20
-30 -40
```

Sample Output 2

```
-1
```

Sample Input 3

```
2 1
-10 -20
-30 -40
```

Sample Output 3

```
4
```

Problem S4: Stakeout

35 Points / Time Limit: 7.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc154s4*

MI6 has received intelligence that members of the evil Spectre organization may be holding a gathering somewhere along a certain street in London. Unfortunately, they're not sure exactly in which building this gathering will take place. As such, they'll need to recruit some agents to stake out the street and look for suspicious acitivity.

The street can be represented by a number line. There are N ($1 \le N \le 300{,}000$) suspicious buildings which must be monitored, with the i-th one at position B_i ($-10^9 \le B_i \le 10^9$).

There are also M ($1 \le M \le 300{,}000$) agents stationed along the street, with the i-th one at position A_i ($-10^9 \le A_i \le 10^9$). All $N + M$ of these positions are distinct. Agent i has a sight range of R_i ($1 \le R_i \le 10^9$), meaning that they're able to keep watch over all buildings at positions in the inclusive range of positions $[A_i - R_i, A_i + R_i]$. Times are tough at MI6, with the agents not even willing to take this assignment for free – M will have to pay agent i a fee of 2^i dollars to stay at their post, otherwise they'll get bored of the stakeout and go out for some steak instead.

In light of this monetary issue, M will need to decide on a tradeoff between cost and thoroughness. A hired agent can successfully keep an eye on all buildings in his range simultaneously, but it might not be sufficient to have each building only be watched by a single agent, in case they're taken out. As such, M would like to ask himself Q ($1 \le Q \le 10$) questions, with the i-th one asking "What's the minimum cost to hire agents such that each suspicious building is in sight of at least C_i ($1 \le C_i \le M$) of them, if this is possible at all?". Can you help M answer each of these questions, for the sake of the MI6 fundraising department? If it's impossible to have each building covered by at least C_i agents, you should output −1 instead. Otherwise, the total cost (in dollars) may be quite large, so you only need to output it modulo $10^9 + 7$.

Subtasks

In test cases worth 4/35 of the points, $N \le 10$ and $M \le 10$.
In test cases worth another 5/35 of the points, $N \le 100$ and $M \le 100$.
In test cases worth another 8/35 of the points, $N \le 2000$ and $M \le 2000$.

Input Format

The first line of input consists of three space-separated integers N, M, and Q.
The next N lines each consist of a single integer B_i, for $i = 1..N$.
The next M lines each consist of two space-separated integers A_i and R_i, for $i = 1..M$.
The next Q lines each consist of a single integer C_i, for $i = 1..Q$.

Output Format

Output Q lines, one integer per line. The i-th line (for $i = 1..Q$) should consist of the answer to the i-th query (modulo $10^9 + 7$) or −1 if it's impossible.

Sample Input

```
2 4 3
10
20
14 5
22 11
0 1
15 5
1
2
3
```

Sample Output

```
6
22
-1
```

Explanation

If M hires agents 1 and 2 (for a cost of $2^1 + 2^2 = 6$ dollars), each building will be in sight of a single agent, which is just enough to satisfy the first question. Hiring agents 1, 2, and 4 will allow each building to be in sight of at least two agents. It's impossible to have each building be in sight of three or more agents.