# WOBURN CHALLENGE

## 2017-18 On-Site Finals

Sunday, May 13th, 2018

*Senior Division Problems*

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# Problem S1: An Interspecific Army

*9 Points / Time Limit: 3.00s / Memory Limit: 64M* *Submit online: http://wcipeg.com/problem/wc17fs1*

The Great Cow-Monkey War of 2017 has left Scarberia in a fragile and unstable state. Interspecies tension is at an all-time high, and both sides are working relentlessly to rebuild their economies, social systems, and militias. Little do the cows and monkeys know that amidst the turmoil, the world is at risk of being taken over by an even more dangerous foe — a group of aliens known as the Party of Extraterrestrial Gangsters (or PEG for short)! PEG has decided that it's the perfect time to divide and conquer Scarberia by exploiting its current internal hostilities. Fortunately, Bo Vine and the Head-Monkey have learned of the impending invasion in time, and an important realization has dawned on them: now is not the time to engage in petty quarrels with one another. All of Scarberia must unite, at least temporarily, to face an even greater evil. The animals have thus formed a truce, and started to plan for the upcoming war!

With an overall battle formation decided, the cows and monkeys are now ready to fight against the incoming alien invaders! …Well, almost. You see, the commanders realized that it is not too effective to simply line up large platoons of uniformly cows or uniformly monkeys. They must work together at a more local scale, with each cow and monkey taking immediate advantage of each others' strengths and weaknesses. As such, the commanders have decided to pair up cows and monkeys to form individual combat units on the battlefield.

Between the united armies, there are $N$ cows and $N$ monkeys ($1 \le N \le 10^5$). The $i$-th cow has a combat skill level of $C_i$ ($1 \le C_i \le 10^6$), while the $i$-th monkey has a combat skill level of $M_i$ ($1 \le M_i \le 10^6$). Each cow is to be paired up with a monkey to form a combat unit, such that there are $N$ combat units total and each of the $2N$animals is a part of exactly one unit.

A combat unit is most effective when the skill levels of its two members don't differ too greatly. More formally, we can define the *combat skill differential* of a given unit to be the absolute difference between the combat skill levels of the two animals making up that unit. For a given pairing of the $2N$ animals, let $D$ denote the maximum combat skill differential across all $N$ combat units. Bo Vine and the Head-Monkey would like to pair up the animals such that $D$ ends up being as small as possible. Can you help them determine the minimum possible value of $D$ which can be achieved?

**Subtasks**

In test cases worth 2/9 of the points, $N \le 10$ and $1 \le C_i, M_i \le 2$ for each $i$.
In test cases worth another 2/9 of the points, $N \le 1000$ and $1 \le C_i, M_i \le 2$ for each $i$.
In test cases worth another 4/9 of the points, $N \le 1000$.

**Input Format**

The first line of input consists of a single integer, $N$.
The second line consists of $N$ space-separated integers, $C_1, \ldots, C_N$.
The third line consists of $N$ space-separated integers, $M_1, \ldots, M_N$.

**Output Format**

Output a single integer, the minimum possible value of $D$ which can be achieved.

**Sample Input**

```
4
5 1 9 1
2 7 4 6
```

**Sample Output**

```
3
```

**Sample Explanation**

One possible optimal arrangement is to pair:

- cow 1 with monkey 2 (a combat skill differential of $|5 - 7| = 2$)
- cow 2 with monkey 1 (a combat skill differential of $|1 - 2| = 1$)
- cow 3 with monkey 4 (a combat skill differential of $|9 - 6| = 3$)
- cow 4 with monkey 3 (a combat skill differential of $|1 - 4| = 3$)

The maximum of these differentials is 3, and it is not possible to go any lower.

# Problem S2: Cowmmunication Network

*13 Points / Time Limit: 4.00s / Memory Limit: 64M* *Submit online: http://wcipeg.com/problem/wc17fs2*

Great military strategists know that a battle occurs beyond just the front lines. To prepare for the the upcoming war against the Party of Extraterrestrial Gangsters, the cows and monkeys will need to have an impeccable communications network for relaying important military commands behind the scenes. To this end, chief scientist of the bovine army Guglielmoo Marcowni has developed a powerful new technology — the radio! While his radio technology is very impressive, it sometimes suffers from the issue of poor signals. To quantify the smoothness of a network, Marcowni has developed a measure of signal quality known as *Communication Compatibility*. A large Communication Compatibility score between radios suggests a great connection, a negative score suggests a very poor connection, and a score of zero suggests a so-so connection.

The cow-monkey army is made up of $N$ ($2 \le N \le 100{,}000$) combat units, numbered from 1 to $N$. Each unit carries a single radio that can communicate with other radios via specific communication channels. In total, there are $M$ ($0 \le M \le 200{,}000$) potential communication channels amongst all of the units. Implementing the $i$-th communication channel would allow units $A_i$ and $B_i$ ($1 \le A_i, B_i \le N$; $A_i \ne B_i$) to radio each other directly, with a Communication Compatibility of $C_i$ ($-10^6 \le C_i \le 10^6$). No two communication channels connect the same unordered pair of combat units.

Bo Vine and the Head-Monkey want to create a communication network out of one or more of the $M$ potential communication channels. The network must be constructed such that every pair of combat units is able to communicate over the network, either directly or indirectly. If a combat unit $i$ can communicate with both combat units $j$ and $k$ (either directly or indirectly), then combat units $j$ and $k$ are also considered able to communicate with each other indirectly.

Bo Vine and the Head-Monkey want to make their network as smooth as possible. Naturally, their choice will be based on the Communication Compability scores of the channels they choose to implement. To help them quantify the overall smoothness of the network, Marcowni has defined a benchmark known as the *Cumulative Communication Compatibility* (CCC) score. The CCC score of the network is defined to be the sum of the Communication Compatibilities of all communication channels that make up the network. Please help Marcowni determine the maximum possible CCC score that a valid network connecting all $N$ combat units could have (note that this value may not fit into a 32-bit signed integer). Unfortunately, it's possible that no such valid network may exist, in which case you should report "`Impossible`" instead.

**Subtasks**

In test cases worth 3/13 of the points, $N \le 10$, $M \le 10$, and $C_i < 0$ for each $i$.
In test cases worth another 3/13 of the points, $N \le 1000$, $M \le 2000$, and $C_i < 0$ for each $i$.
In test cases worth another 6/13 of the points, $N \le 1000$, $M \le 2000$.

**Input Format**

The first line of input consists of two space-separated integers, $N$ and $M$.
$M$ lines follow, the $i$-th of which consists of three space-separated integers, $A_i$, $B_i$, and $C_i$, for $i = 1..M$.

**Output Format**

Output a single line consisting of either a single integer, the maximum possible CCC score of any valid network, or the string "`Impossible`" if no valid network exists.

**Sample Input 1**

```
4 5
1 2 -1
2 3 -5
3 4 -3
4 1 -2
4 2 -3
```

**Sample Output 1**

```
-6
```

**Sample Input 2**

```
5 4
1 2 5
2 3 2
3 1 -1
4 5 0
```

**Sample Output 2**

```
Impossible
```

**Sample Explanation**

In the first example, one possible optimal network is by implementing the first, third, and fourth communication channels. The CCC score of such a network is $(-1) + (-3) + (-2) = -6$.

In the second example, there is no way to allow any of the first three combat units to communicate with any of the last two units, so it is impossible to build a valid network.

*Note: This statement has been altered from the version presented on the live contest to eliminate uncertainty regarding the strength of test data in the original. In particular, the bound on N has been reduced from 200,000 to 200. We apologize for the issue!*

# Problem S3: Explosive Ordinance Disposal

*17 Points / Time Limit: 4.00s / Memory Limit: 64M* *Submit online: http://wcipeg.com/problem/wc17fs3*

The Party of Extraterrestrial Gangsters has begun its invasion of Earth! Vast armies of PEG soldiers have been deployed down to the surface throughout Scarberia, and the cows and monkeys have engaged them in battle.

Amidst the fighting, however, the aliens have also transported something else to the planet's surface — a bomb with devastating nuclear power! All life in Scarberia, and perhaps the rest of Earth, would surely cease if the bomb were to detonate. Fortunately, the PEG leaders are honourable enough to give their enemies a fighting chance. As such, they've set the bomb to go off after a period of three hours, and implanted a system for defusing it. They've even included an instruction manual along with it!

On the surface of the bomb, there are $N$ ($1 \le N \le 200$) electrical terminals. There are also $N - 1$ wires running amongst the terminals, the $i$-th of which runs between terminals $A_i$ and $B_i$ ($1 \le A_i, B_i \le N$), and is either black (if $C_i = 0$), or is otherwise white (if $C_i = 1$). The wires have been arranged such that all pairs of terminals are reachable from one another by following a sequence of wires.

Bo Vine and the Head-Monkey have gotten their hands on the bomb and its accompanying instruction manual. According to the manual, the bomb will turn itself off if the following conditions are all met:

1. Each terminal $i$ receives an electrical current with some voltage $V_i$, such that $V_i$ is a positive integer.
2. For each black wire $i$ (such that $C_i = 0$), the greatest common divisor (GCD) of $V_{Ai}$ and $V_{Bi}$ is equal to 1.
3. For each white wire $i$ (such that $C_i = 1$), the GCD of $V_{Ai}$ and $V_{Bi}$ is greater than 1.

Bo Vine has ordered his cow engineers to prepare the necessary electrical equipment as quickly as possible. Meanwhile, the Head-Monkey has personally taken it upon herself to come up with a set of voltages $V_{1..N}$ which will successfully satisfy the conditions to defuse the bomb. However, having realized that PEG is essentially mocking them by dispatching a bomb which may be defused so easily, she's decided to get back at them by demonstrating the monkeys' superior intelligence and successfully defusing the bomb using as little voltage as possible. Help the Head-Monkey determine the minimum possible total voltage (sum of $V_{1..N}$ values) required to get the job done. Just make sure to figure it out within three hours!

**Subtask**

In test cases worth 3/17 of the points, $C_1 = C_2 = \ldots = C_{N-1}$.

**Input Format**

The first line of input consists of a single integer, $N$.
$N - 1$ lines follow, the $i$-th of which consists of three space-separated integers, $A_i$, $B_i$, and $C_i$, for $i = 1..(N - 1)$.

**Output Format**

Output a single integer, the minimum possible total voltage required to defuse the bomb.

**Sample Input**

```
7
4 1 1
4 5 0
7 6 1
3 6 1
1 7 0
2 4 1
```

**Sample Output**

```
16
```

**Sample Explanation**

It's optimal to send:

- a 1-volt current to terminal 5;
- 2-volt currents to terminals 1, 2, and 4; and
- 3-volt currents to terminals 3, 6, and 7.

# Problem S4: Ultimatum

*26 Points / Time Limit: 5.00s / Memory Limit: 64M* *Submit online: http://wcipeg.com/problem/wc17fs4*

The Party of Extraterrestrial Gangsters are not at all pleased with how their invasion of Earth is going so far. Their soldiers on the ground are losing in combat to the well-prepared battalions of cows and monkeys, and their nuclear bomb has been defused with particular ease. Fortunately for them, PEG's space battleship is also outfitted with an enormously powerful vaporizer beam capable of wiping out Scarberia in an instant. The PEG leaders were hoping to avoid resorting to such a merciless tactic, but they're running out of alternatives. However, before proceeding with any vaporization, they'll issue an ultimatum to the Scarberian leaders, detailing the amount of potential destruction and providing an opportunity for unconditional surrender instead.

Downtown Scarberia has a row of $N$ ($1 \le N \le 200{,}000$) skyscrapers, numbered 1 to $N$ from left to right. Skyscraper $i$ has a height of $H_i$ stories ($1 \le H_i \le 10^9$), and is occupied by $C_i$ citizens ($1 \le C_i \le 10^9$).

The PEG leaders have come up with a list of $M$ ($1 \le M \le 200{,}000$) possible plans of attack using their vaporizer beam. The $i$-th plan is defined by three integers, $X_i$, $L_i$, and $R_i$ ($0 \le X_i \le N$; $1 \le L_i \le R_i \le N$). It consists of two phases, as follows:

1. The beam will lock onto the tallest remaining un-vaporized skyscraper, and vaporize it. If there are multiple such skyscrapers with equal heights, they'll choose one of them uniformly at random. They'll repeat this process $X_i$ times, meaning that exactly $X_i$ skyscrapers will get vaporized in this phase.
2. The beam will vaporize all of the un-vaporized skyscrapers numbered between $L_i$ and $R_i$, inclusive. Note that some of these skyscrapers may have already been vaporized in phase 1 of the attack, meaning that anywhere between 0 and $R_i - L_i + 1$ skyscrapers (inclusive) will get vaporized in this phase.

For each possible plan of attack, the PEG leaders are interested in the expected number of citizens who would get vaporized in it (that is, the expected total number of citizens occupying all of the vaporized skyscrapers). Disclosing these statistics in an ultimatum would be sure to drive the Scarberian forces to immediately surrender! Help them determine this value for each of the $M$ plans of attack. Note that none of the plans will actually be executed (yet), meaning that no vaporization will carry over between them.

**Subtasks**

In test cases worth 7/26 of the points, $N \le 2000$, $M \le 2000$, and $H_i = 1$ for each $i$.
In test cases worth another 6/26 of the points, $N \le 2000$ and $M \le 2000$.

**Input Format**

The first line of input consists of a single integer, $N$.
$N$ lines follow, the $i$-th of which consists of two space-separated integers, $H_i$ and $C_i$, for $i = 1..N$.
The next line consists of a single integer, $M$.
$M$ lines follow, the $i$-th of which consists of three space-separated integers, $X_i$, $L_i$, and $R_i$, for $i = 1..M$.

**Output Format**

Output *M* lines, the *i*-th of which contains a single real number, the expected number citizens who would get vaporized in the *i*-th plan of attack. Your answer must have at most $10^{-5}$ absolute or relative error compared to the judge's answer to be considered correct.

**Sample Input**

```
5
4 1
2 50
6 400
4 100
3 33
3
5 1 5
4 2 2
2 2 4
```

**Sample Output**

```
584.0
584.0
550.5
```

**Sample Explanation**

In the first plan of attack, all 5 skyscrapers would already get vaporized in the first phase.

In the second plan of attack, the 4 tallest skyscrapers would get vaporized in the first phase, leaving just skyscraper 2 behind, which would happen to get targeted in the second phase anyway.

In the third plan of attack, phase 1 would involve vaporizing skyscraper 3 followed by either skyscraper 1 or 4, with 50% probability each. In the end, either 550 or 551 citizens would be vaporized, with 50% probability each (depending on the skyscraper chosen in phase 1).

# Problem S5: Crop Rectangles

*35 Points / Time Limit: 6.00s / Memory Limit: 128M* *Submit online: http://wcipeg.com/problem/wc17fs5*

After weeks of gruelling combat, the unified cow-monkey army has been abruptly called home. Bo Vine and the Head-Monkey have both realized that continuing to fight this war against the Party of Extraterrestrial Gangsters is senseless — if the aliens are provoked into using their devastating vaporization beam, Scarberia is surely doomed. However, they're not prepared to just surrender the Earth to PEG either. Perhaps an alternative resolution can be found? Perhaps an offer of peace can be extended to PEG?

Upon consultation with his leading extraterrestrial linguistics experts, Bo Vine has devised a plan for communicating with the aliens — using the universal alien language of crop formations! Crop circles are difficult to create, but crop rectangles will do just as well. In order to convey a message of peace, Bo Vine has determined that a sequence of $N$ ($1 \le N \le 20$) crop rectangles will be necessary, the $i$-th of which should be $R_i$ metres tall and $C_i$ metres wide ($1 \le R_i, C_i \le 80$) when viewed from above.

Unfortunately, having spent just about all of their resources on the war against PEG, the cows and monkeys are having a hard time finding any equipment to mow down crops. The Head-Monkey's old lawnmower will have to do. The lawnmower is able to mow down a 1m×1m square of crops at a time. So, for each crop rectangle $i$, Bo Vine has drawn out a grid of 1m×1m cells in a field of crops, with $R_i$ rows and $C_i$ columns. What remains is for the Head-Monkey to simply drive the lawnmower around and mow down the crops in all grid's cells! They'll intend to repeat this process independently for all $N$ crop rectangles.

For each crop rectangle, the lawnmower will need to start in the bottom-left corner of the grid, facing in any cardinal direction of the Head-Monkey's choice. After that, at any point, the Head-Monkey may either drive the lawnmower forwards by one cell (in the direction it's currently facing), or turn it 90 degrees in either direction (either clockwise or counter-clockwise). Driving it forwards by one cell consumes one litre of gas, while turning it doesn't consume any.

Much to the Head-Monkey's dismay, her old lawnmower doesn't handle as well as she seemed to remember. In particular, its turning radius is quite lacking, resulting in her needing to drive it forwards at least twice between any two consecutive turns. For example, she may not turn → turn or turn → drive → turn at any point, but she may turn → drive → drive → turn.

For each crop rectangle $i$, the Head-Monkey will need to keep driving the lawnmower around until it has visited each of the $R_i \times C_i$ cells at least once, thus mowing down crops in the required shape. Note that cells may be driven through multiple times if necessary. The lawnmower must never leave the boundaries of the grid, as that would cause the crop formation to no longer be accurate. Unfortunately, the lawnmower's poor turning radius may result in certain crop rectangles being impossible to create altogether.

The Head-Monkey also doesn't want to risk running out of gas before the crop formations are complete, lest they accidentally send a message of war to PEG instead! As such, she'd like to minimize the amount of gas used. For each independent crop rectangle, help her determine the minimum amount of gas required to create it, or report a value of $-1$ if it's impossible to do so.

**Subtasks**

In test cases worth 4/35 of the points, $1 \leq R_i, C_i \leq 4$ for each $i$.
In test cases worth another 8/35 of the points, $1 \leq R_i, C_i \leq 6$ for each $i$.

**Input Format**

The first line of input consists of a single integer, $N$.
$N$ lines follow, the $i$-th of which consists of two space-separated integers, $R_i$ and $C_i$, for $i = 1..N$.

**Output Format**

Output $N$ lines, the $i$-th of which consists of a single integer, the minimum amount of gas (in litres) required to create the $i$-th crop rectangle, or $-1$ if it's impossible

**Sample Input**

```
3
3 1
3 2
1 1
```

**Sample Output**

```
2
-1
0
```

**Sample Explanation**

For the first rectangle, assuming that the lawnmower starts in the bottom-left corner of the grid facing upwards, it can simply drive forwards twice to mow down the remaining two cells.

For the second rectangle, it's possible to mow down up to 5 different cells — if the lawnmower starts facing to the right, it can drive forwards, turn counter-clockwise (to face upwards), drive forwards twice, turn counter-clockwise (to face to the left), and drive forwards once. However, it's impossible to mow down all 6 cells in the grid.