WOBURN CHALLENGE

2017-18 Online Round 2

Friday, February 23rd, 2018

Senior Division Problems

Automated grading is available for these problems at:
wcipeg.com

For more problems from past contests, visit:
woburnchallenge.com

Problem S1: Keeping Score

14 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc172s1*

There's nothing like a bit of friendly competition, even when your life is on the line! Legolas and Gimli have taken to counting how many enemies they're each able to kill in each confrontation, in an effort to one-up one another.

During the battle of Helm's Deep, Legolas killed L ($2 \le L \le 200{,}000$) enemies, the i-th of which had a strength level of SL_i ($1 \le SL_i \le 10^9$). Meanwhile, Gimli killed G ($1 \le G < L$) enemies, the i-th of which had a strength level of SG_i ($1 \le SG_i \le 10^9$).

Though Gimli killed fewer enemies than Legolas did, he's not about to admit defeat to the elf so easily. As such, he's gotten the idea to introduce a new rule: "All enemies with strength levels smaller than X don't count" (for some positive integer X no larger than 10^9). Help Gimli find any value of X which would cause him to "win" (in other words, such that Gimli killed strictly more enemies with strength levels greater than or equal to X than Legolas did). If no possible value of X would have this result, output −1 instead.

Subtasks

In test cases worth 7/14 of the points, $L \le 1000$.

Input Format

The first line of input consists of two space-separated integers, L and G.
L lines follow, the i-th of which consists of a single integer SL_i (for $i = 1..L$).
G lines follow, the i-th of which consists of a single integer SG_i (for $i = 1..G$).

Output Format

Output a single integer, any valid value of X which would cause Gimli to win, or −1 if there's no such value.

Sample Input 1

```
5 4
84
6
105
54
30
91
84
28
66
```

Sample Output 1

```
60
```

Sample Input 2

```
2 1
3 3
3
```

Sample Output 2

```
-1
```

Sample Explanation 2

In the first case, if $X = 60$, then Legolas's score will be 2 (with his 1st and 3rd killed enemies counting), while Gimli's score will be 3. Note that there exist other valid values of X which would also be accepted.

In the second case, if $X \le 3$, then both Legolas's score will be 2 and Gimli's will be 1. If $X > 3$, then both scores will be 0. Either way, Gimli can't win.

Problem S2: Don’t Follow the Lights

22 Points / Time Limit: 6.00s / Memory Limit: 256M *Submit online: http://wcipeg.com/problem/wc172s2*

Led by the creature Gollum, Frodo and Sam have set out to sneak their way into Mordor. Their journey takes them through the Dead Marshes, a mysterious, ancient battleground which has a way of leaving travellers lost until they perish.

The Dead Marshes can be represented as a 2D grid, with R rows and C columns ($2 \le R, C \le 1500$). The three travellers seek to find their way from a certain starting cell to a destination one. Some of the cells may contain torches, while each other cell appears to be empty, but may contain dangerous boggy water – it's hard to tell which ones are in fact safe to walk on. Each cell is described by one of four characters:

- "`S`": The starting cell, which is otherwise empty (there's exactly one such cell)
- "`D`": The destination cell, which is otherwise empty (there's exactly one such cell)
- "`.`": An empty cell
- "`*`": A cell with a torch

The party would like to reach the destination cell from the starting one as quickly as possible. Every minute, they may move from their current cell to an adjacent one (either up, down, left, or right). They may not move outside the grid, and may not move into a cell containing a torch.

However, empty cells may be dangerous as well, so they've agreed to also pay attention to the warning words uttered by Gollum: "Don't follow the lights". To be precise, this means that they may not move in a given direction if there are at least two cells containing torches further ahead in that direction, in that same row/column. For example, if $C = 6$ and a certain row has torches in its 3rd and 6th cells, then the party may not move right from the 1st cell to the 2nd cell in that row, but they may move left from the 2nd cell to the 1st cell, or left/right between the 4th and 5th cells.

Please help Frodo, Sam, and Gollum determine the minimum amount of time required for them to reach the destination cell while following the above rules. Output −1 instead if they can't make it at all, and are doomed to wander the Dead Marshes forever.

Subtasks

In test cases worth 18/22 of the points, $R \le 100$ and $C \le 100$.

Input Format

The first line of input consists of two space-separated integers, R and C.
R lines follow, the i-th of which consists of characters representing the i-th row of the grid, for $i = 1..R$.

Output Format

Output a single integer, the minimum amount of time required to reach the destination cell, or −1 if it's impossible to do so.

Sample Input 1

```
6 5
**...
.*...
.D.*.
..*..
.....
*..S.
```

Sample Output 1

```
7
```

Sample Input 2

```
2 10
*....*.D.*
.S.*....*.
```

Sample Output 2

```
-1
```

Sample Explanation 2

In the first case, one optimal path is indicated below:

```
**...
.*...
6D.*.
5.*..
432..
*.1S.
```

In the second case, they would need to move up into the 1st row to get around the torch in the 4th cell of the 2nd row. However, moving rightwards in the 1st row towards the torch in the 6th cell is then forbidden, due to the presence of the torch in the 10th cell as well.

Problem S3: Battle of the Pelennor Fields

27 Points / Time Limit: 6.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc172s3*

An enormous, decisive battle is about to take place on the Pelennor Fields before the city of Minas Tirith! A noble army of Gondorian men will engage in battle against Sauron's evil army of orcs in an attempt to save their home and all of Middle Earth. Gandalf has given you your own vital task – not to actually participate in combat, but to report to him the state of the battle as it unfolds.

The battlefield can be represented as an infinite number line, and is initially empty. N ($1 \le N \le 300{,}000$) events will then occur over the course of the battle, one after another. The i-th event's type is indicated by the value E_i ($1 \le E_i \le 2$):

- If $E_i = 1$, then an orc arrives on the battlefield at position O_i ($0 \le O_i \le 10^9$).
- Otherwise, if $E_i = 2$, then a Gondorian archer arrives on the battlefield at position A_i ($0 \le A_i \le 10^9$), and having a bow range of R_i ($1 \le R_i \le 10^9$). Such an archer is able to shoot any orcs which are at most R_i units of distance away from A_i.

All N combatants' positions are distinct.

An orc is "vulnerable" if there's at least one archer on the battlefield who is able to shoot that orc. After each of the N events, you'd like to report to Gandalf the number of orcs currently on the battlefield which are not vulnerable.

Subtasks

In test cases worth 4/27 of the points, $N \le 1000$ and $R_i = 1$ for each applicable i.
In test cases worth another 4/27 of the points, $R_i = 1$ for each applicable i.
In test cases worth another 8/27 of the points, $R_i = 10^6$ for each applicable i.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of a single integer, E_i, followed by either 1 more integer O_i (if $E_i = 1$) or 2 more integers A_i and R_i (if $E_i = 2$), for $i = 1..N$.

Output Format

Output N lines, the i-th of which should consist of a single integer, the number of non-vulnerable orcs on the battlefield after the first i events.

Sample Input

```
7
1 15
1 22
2 18 3
1 19
2 5 12
1 23
1 0
```

Sample Output

```
1
2
1
1
1
2
2
```

Sample Explanation

After the 2nd event, there are still no archers on the field, meaning that both orcs are non-vulnerable. After the 3rd event, the newly-arrived archer is just barely able to shoot the first orc, while the second orc is too far away and so is still non-vulnerable. After the 7th event, the orcs at positions 22 and 23 are still non-vulnerable, while the remaining 3 orcs are vulnerable.

Problem S4: One Does Not Simply Walk Into Mordor

37 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc172s4*

Having walked many a mile over the course of their journey, and with the last of their strength just about depleted, Frodo and Sam have at last changed their mind about the whole "walking into Mordor to throw the Ring into Mount Doom" plan. It really does seem rather dangerous! Fortunately, they've devised an alternate strategy to destroy the One Ring, thereby saving Middle Earth from Sauron's impending rule.

The Ring is a circle-shaped object with N ($3 \le N \le 400$) evenly-spaced markings inscribed on it. Numbering the markings in clockwise order, the i-th marking is an inscription of the integer L_i ($1 \le L_i \le 10^9$) in Sauron's Black Speech. For the sake of science, we can consider the ring to be arbitrarily thin (a circle on a plane, when viewed from above), and the markings to be arbitrarily small (points on that circle).

The Ring is normally resistant to simply being cut into pieces with a sword, but Frodo and Sam have realized the secret behind its power. They're going to simultaneously make one or more precise cuts through it. Each cut can be thought of as a line segment intersecting the circle at two points. No cut may pass directly through any of the Ring's markings.

The Ring is only able to keep itself together if there exists at least one pair of markings i and j, such that $L_i \ne L_j$ and none of the cuts intersect the line segment connecting those two markings. As such, Frodo and Sam hope to perform their cuts so as to interrupt the power of all of these pairs of markings, thus destroying the Ring.

Performing many cuts simultaneously is a difficult task for a pair of Hobbits, so they'd like to minimize the number of cuts. There's just one detail they can't decide on: Frodo is confident that they'll be able to execute any set of cuts they'd like to, while Sam is concerned that it'll only be doable if none of the cuts' line segments intersect with one another.

So, let's consider two different cases – either any set of possibly-intersecting cuts may be made, or the cuts must all be non-intersecting. For each case, determine the minimum number of cuts which must be made in order for the Ring to be destroyed. At least one cut is guaranteed to be required (in other words, the integers $L_{1..N}$ will not all be equal).

Subtasks

In test cases worth 6/37 of the points, $N \le 15$.
In test cases worth another 16/37 of the points, $N \le 50$.

Input Format

The first line of input consists of a single integer N.
The next line consists of integers $L_{1..N}$.

Output Format

Output two space-separated integers, the minimum number of cuts which must be made with and without them being allowed to intersect, respectively.

Sample Input

```
8
1 2 4 4 1 1 3 2
```

Sample Output

```
3 4
```

Sample Explanation

The below two diagrams illustrate sample ways in which the ring can be destroyed with as few cuts as possible, with and without the cuts being allowed to intersect: